RIGHT TRIANGLE TRIGONOMETRY APPLICATIONS
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Before we begin some examples let us review some important right triangle trigonometry definitions and right triangle properties.

The common method for labeling a right triangle is; label the acute angles A and B, label the right angle C, and label the sides opposite each angle with the same lower case letter. See the diagram below.

\[ \text{NOTE: The hypotenuse is labeled } c \text{ and the two legs of the right triangle are labeled } a \text{ and } b. \]

Below are the important right triangle trigonometry definitions. For acute angles A and B.

**Acute Angle A.**

\[ \sin(A) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \]

\[ \cos(A) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \]

\[ \tan(A) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \]

**Acute Angle B.**

\[ \sin(B) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{b}{c} \]

\[ \cos(B) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{a}{c} \]

\[ \tan(B) = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{a} \]
Some additional useful right triangle properties are;

1. The sum of the acute angles in a right triangle equals 90 degrees.

   That is, \( A + B = 90^\circ \Rightarrow \begin{align*}
   A &= 90 - B \\
   B &= 90 - A
   \end{align*} \)

2. The famous pythagorean theorem.

   The hypotenuse squared equals the sum of the squares of the two legs.

   That is, \( c^2 = a^2 + b^2 \Rightarrow \begin{align*}
   a &= \sqrt{c^2 - b^2} \\
   b &= \sqrt{c^2 - a^2}
   \end{align*} \)

**EXAMPLES:**

In each of the following examples the angle measures are rounded to the nearest tenth of a degree and the lengths of the sides are rounded to three significant digits.

**Example 1.**

A 24 foot ladder leaning against the side of a house makes a 70° angle with the ground. How far up the side of the house does the ladder reach?

You have probably heard the old saying, a picture (diagram) is worth a thousand words. This statement is certainly true when doing right triangle trigonometry applications. By drawing a diagram, we gain a greater understanding of the problem. The diagrams for right triangle trigonometry applications are all right triangles.

![Diagram](image)

Let \( h \) = Distance from the top of the ladder to the base of the building.
Side $h$ is opposite the $70^\circ$ angle and we know the hypotenuse is 24. What right triangle trigonometry definition uses the opposite side and hypotenuse? The sine definition.

By definition, $\sin(70^\circ) = \frac{\text{opposite side}}{\text{hypotenuse}}$

By substitution, $\sin(70^\circ) = \frac{h}{24}$

To solve for $h$, multiply both sides by 24.

$$\frac{24}{1} \cdot \sin(70^\circ) = \frac{h}{24} \cdot \frac{24}{1}$$

$h = 24 \sin(70^\circ)$ use your calculator.

$h = 22.6 \text{ ft.}$

Hence, the ladder reaches 22.6 ft. up the side of the building.

**Example 2.**

A support guy wire is stretched from a broadcasting tower at a point 200 feet above ground to an anchor 110 feet from the base of the tower. If three guy wires are needed to anchor the tower, then how much wire is needed?

Don't forget that old saying, a diagram is worth a thousand words.

**Diagram**

![Diagram of a tower and guy wire]

Let $L =$ length of one of the three guy wires.

Let $A =$ The angle formed between the guy wire and the ground.

Since the 110 ft. side is adjacent to angle $A$ and the 200 ft. side is opposite angle $A$, we will use the tangent definition.
By definition, \[ \tan(A) = \frac{\text{opposite side}}{\text{adjacent side}} \]

By substitution, \[ \tan(A) = \frac{200}{110} \]
\[ \tan(A) = 1.8182 \]
\[ A = \tan^{-1}(1.8182) \] use your calculator
\[ A = 61.2^\circ \]

Hence, angle \( A \) is \( 61.2^\circ \).

To find the length \( L \) we can now use either the sine definition or the cosine definition. Take your pick.

By definition, \[ \cos(A) = \frac{\text{adjacent side}}{\text{hypotenuse}} \]

By substitution, \[ \cos(61.2^\circ) = \frac{110}{L} \]

Solve for \( L \), \[ L = \frac{110}{\cos(61.2)} \]
\[ L = \frac{110}{0.4818} \]
\[ L = 228 \text{ ft.} \]

Again, use your calculator.

The length of the guy wire is 228 ft.

Hence, the length of three guy wires is \( 3(228 \text{ ft.}) = 684 \text{ ft.} \).

**NOTE:**

The above example can also be solved using the Pythagorean Theorem. I used the method which illustrated the use of the right triangle trig. definitions. For comparison, you might want to solve it using the Pythagorean Theorem. Who knows, it might be fun-and Pythagoras would be proud of you!
Example 3.

A ramp 16 feet long rises to a loading platform that is 3 feet off the ground. Find the angle, \( \theta \), that the ramp makes with the ground.

Draw a diagram.

Let \( \theta \) = the angle the ramp makes with the ground.

Since the 3 ft. side is opposite the angle \( \theta \) and the 16 ft. side is the hypotenuse, we will use the \( \text{sine} \) definition.

By definition, \( \sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}} \)

By substitution, \( \sin(\theta) = \frac{3}{16} \)

\( \sin(\theta) = .1875 \)
\( \theta = \sin^{-1}(0.1875) \) [Use your calculator.]
\( \theta = 10.8^\circ \)

Hence the angle the ramp makes with the ground is \( 10.8^\circ \).

Example 4.

A surveyor wants to know the width of a river. From point \( C \) directly across the river from point \( B \) she walks 150 feet downstream to point \( A \). She measures angle \( \text{CAB} \) to be \( 54^\circ \). How wide is the river?
Draw a diagram.

Let $W$ = the width of the river.

Since side $W$ is opposite the angle and the 150 ft. side is adjacent to the angle, we will use the tangent definition.

By definition, \[ \tan(54^\circ) = \frac{\text{opposite side}}{\text{adjacent side}} \]

By substitution, \[ \tan(54^\circ) = \frac{W}{150} \]

\[ W = 150 \tan(54^\circ) \]

\[ W = 206 \text{ ft.} \]

Hence, the river is 206 feet wide.

In the next two examples the terms angle of elevation and angle of depression are used.

Angle of elevation is defined as an angle measured above a horizontal reference line.

Angle of depression is defined as an angle measure below a horizontal reference line.

Again, a diagram is worth a thousand words.
Example 5.

From a point 65 feet from the base of a tree, the angle of elevation to the top of the tree is $35.6^\circ$. Find the height of the tree.

Draw a diagram.

Let $h$ = the height of the tree

Since the 65 ft. side is adjacent to the angle and side $h$ is opposite the angle, we will use the *tangent* definition.

By definition, \[ \tan(35.6^\circ) = \frac{\text{opposite side}}{\text{adjacent side}} \]

\[ \tan(35.6^\circ) = \frac{h}{65} \]

\[ h = 65 \tan(35.6^\circ) \]

\[ h = 46.5 \text{ ft.} \]

Hence, the height of the tree is 46.5 feet.
Example 6.

From a 125 foot observation tower on the coast, an observer sights a boat. The angle of depression of the boat is 6°. How far is the boat from the coast?

Draw a diagram.

Let \( d \) = distance from the boat to the coast.

**NOTE:** The angle inside the right triangle at the base where the boat is at is also 6°. Why? I am glad I asked why. The reason comes from an old property in geometry which says that if two parallel lines are cut by a transversal then alternate interior angles are equal.

By the tangent definition,

\[
\tan(6°) = \frac{\text{opposite side}}{\text{adjacent side}}
\]

\[
\tan(6°) = \frac{125}{d}
\]

\[
d \tan(6°) = 125
\]

\[
d = \frac{125}{\tan(6°)}
\]

\[
d = 1190 \text{ ft.}
\]

Hence, the boat is 1190 feet from the coast.
Directions: Round off the angles to the nearest tenth of a degree and the lengths of the sides to three significant digits.

Exercises: (Hint: First draw and label the right triangle diagram for each problem)

1. A 30 foot ladder leaning against the side of a house makes a 68° angle with the ground. How far up the side of the house does the ladder reach?

2. Find the angle between the diagonal and base of a rectangle with length 12 cm. and width 7 cm.

3. To find the distance across a marsh a surveyor determines two points A and B on opposite sides of the marsh. She then measures a 50 meter perpendicular distance from point B to point C and measures angle BCA to be 53°. What is the distance across the marsh from point B to point A?

4. A ladder 18 feet long leans against a building. The ladder forms an angle of 65° with the ground. Find the horizontal distance from the foot of the ladder to the base of the building.

5. In designing a new building, a ramp for the disabled is to be built from the base of the doorway to level ground. Determine the length of the ramp if the ramp makes an angle of 5.5° with the ground and the base of the doorway is 3 feet above the ground.

6. A lamp post casts a shadow 8 feet long when the angle of elevation of the sun is 64°. How tall is the lamp post?

7. An airplane traveling at 255 mph is descending at an angle of depression of 5.5°. What is the change in altitude of the plane after 8 minutes.

8. From the top of a cliff 146 meters high, the angle of depression of a boat is 23.7°. How far is the boat from the base of the cliff?

9. The Empire State Building is 1250 feet tall. What is the angle of elevation of the top of this building from a point on the ground \( \frac{1}{2} \) mile (1 mile = 5280 ft.) from the base of the building?

10. From a point on ground level the angle of elevation to the top of a mountain is 38°. Then you walk 250 meters farther away from the mountain and find the angle of elevation to be 22°. Find the height of the mountain.

11. If the angle of elevation of the sun is 42°, what is the length of the shadow cast on the ground of a man who is 6 ft. 2 in. tall?

12. From the top of a building 60 feet high, the angle of elevation of the top of a pole is 14°. At the bottom of the building the angle of elevation of the top of the pole is 28°. Find the distance from the pole to the building.
Solutions to the odd-numbered problems and the answers to the even-numbered problems.

1. Remember to first draw a diagram.

Let \( h \) = The distance from the top of the ladder to the base of the building.

By the sine definition,

\[
\sin(68^\circ) = \frac{\text{opposite side}}{\text{hypotenuse}}
\]

\[
\sin(68^\circ) = \frac{h}{30}
\]

\( h = 30 \sin(68^\circ) \)

\( h = 27.8 \text{ ft.} \)

Hence, the ladder reaches 27.8 ft. up the side of the building.

2. The angle between the diagonal and base of the rectangle is 30.3°
3. Draw a diagram.

Let \( d \) = Distance across the marsh from point B to point A.

By the tangent definition,

\[
\tan(53^\circ) = \frac{\text{opposite side}}{\text{adjacent side}}
\]

\[
\tan(53^\circ) = \frac{d}{50}
\]

\[ d = 50 \tan(53^\circ) \]

\[ d = 66.4 \text{ meters} \]

Hence, the distance across the marsh is 66.4 meters.

4. Hence, the foot of the ladder is 7.61 feet from the base of the building.

5. Draw a diagram
By the sine definition,

\[ \sin(5.5^\circ) = \frac{3}{L} \]

\[ L \sin(5.5^\circ) = 3 \]

\[ L = \frac{3}{\sin(5.5^\circ)} \]

\[ L = 31.3 \text{ ft.} \]

Hence, the length of the ramp is 31.3 ft.

6. Hence, the lamp post is 16.4 feet tall.

7. Draw a diagram.

Let \( h \) = the change in altitude of the plane.

First, find the distance the plane travels in 8 min.

\[ D = RT \]

\[ D = (255) \left( \frac{8}{60} \right) \text{ Note: 8 min. } = \frac{8}{60} \text{ hr.} \]

\[ D = 34 \text{ miles.} \]

The distance 34 miles is the hypotenuse of the right triangle.
By the sine definition,

\[
\sin(5.5^\circ) = \frac{\text{opposite side}}{\text{hypotenuse}}
\]

\[
\sin(5.5^\circ) = \frac{h}{34}
\]

\[
h = 34 \sin(5.5^\circ)
\]

\[
h = 3.26 \text{ miles}
\]

Hence, the change in altitude of the plane is 3.26 miles.

8. The boat is 333 meters from the base of the cliff.

9. Draw a diagram.

![Diagram](image)

**NOTE:**

1 mile = 5280 ft.

\[
\frac{1}{2} \text{ mile} = \frac{1}{2}(5280 \text{ ft})
\]

\[
\frac{1}{2} \text{ mile} = 2640 \text{ ft.}
\]

Let \( \theta \) = the angle of elevation.

By the tangent definition,
\[ \tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}} \]

\[ \tan(\theta) = \frac{1250}{2640} \]

\[ \tan(\theta) = .4735 \]

\[ \theta = \tan^{-1}(.4735) \]

\[ \theta = 25.3^\circ \]

Hence, the angle of elevation to the top of the Empire State Building is 25.3\(^\circ\).

10. The height of the mountain is 202 meters.

11. Draw a diagram.

Let L = the length of the man's shadow.

By the tangent definition,

\[ \tan 42^\circ = \frac{\text{opposite side}}{\text{adjacent side}} \]

\[ \tan 42^\circ = \frac{74}{L} \]

\[ L \tan 42^\circ = 74 \]
\[ L = \frac{74}{\tan 42^\circ} \]

\[ L = 82.2 \text{ in. or 6 ft. 10 in.} \]

Hence, the length of the man's shadow is 6 ft. 10 in.

12. The distance from the pole to the building is 212 feet.