MATH 175: Chapter 2 Review: Functions and Their Graphs

In order to prepare for a test including Chapter 2, you need to understand and be able to work problems involving the following topics:

A. Can you determine if a relation is a function? (2.1)

Ex1) Determine if the following relations represent a function. For each function, state the domain and the range.

a. \( \{(4,5),(1,3),(4,2)\} \)  
b. \( \{(3,5),(1,3),(2,5)\} \)

B. Can you determine if an equation is a function? (2.1)

Ex2) Determine if the following equations represent a function. For each function, state its domain. Use interval notation.

a. \( y = x^2 + 1 \)  
b. \( x + y^2 = 1 \)

C. Can you evaluate functions at specific values? (2.1)

Ex3) Let \( f(x) = \frac{x^2 - 1}{x+4} \), find the following:

a. \( f(-2) \)  
b. \( f(-x) \)  
c. \( f(x-2) \)  
d. \( f(x+h) \)

D. Can you find domains of function? (2.1)

Ex4) Find the domains of the following functions. Use interval notation.

a. \( f(x) = \sqrt{3x-12} \)  
b. \( f(x) = \frac{2x}{x^2 - 4} \)  
c. \( f(x) = \frac{x}{\sqrt{x-4}} \)  
d. \( f(x) = \frac{2x}{x^2 + 4} \)
E. Can you find sums, differences, product and quotients of two functions and their domains, given their equations? (2.1)

Ex5) Let \( f(x) = x - 3 \) and \( g(x) = x^2 - 9 \).

a. Find \((f + g)(x)\) and its domain. Use interval notation for the domain.

b. Find \((f - g)(x)\) and its domain. Use interval notation for the domain.

c. Find \((fg)(x)\) and its domain. Use interval notation for the domain.

d. Find \(\frac{f}{g}(x)\) and its domain. Use interval notation for the domain.

Ex6) Let \( f(x) = \frac{5x}{2x-1} \) and \( g(x) = \frac{6x+2}{2x-1} \).

a. Find \((f + g)(x)\) and its domain. Use interval notation for the domain.

b. Find \((f - g)(x)\) and its domain. Use interval notation for the domain.

c. Find \((fg)(x)\) and its domain. Use interval notation for the domain.

d. Find \(\frac{f}{g}(x)\) and its domain. Use interval notation for the domain.

F. Can you determine whether a function is even, odd, or neither both algebraically and graphically? (2.3)

Ex7) Determine graphically if the functions below are even, odd or neither. Explain.

a. 

b. 

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Ex8) Determine algebraically if the functions below are even, odd or neither. Explain.

a. \( f(x) = 3x^4 - 5x^2 \)

b. \( f(x) = 5x^3 - x + 2 \)

c. \( f(x) = \frac{x^2 - 3}{x} \)

G. Can you find the local minima and local maxima of a function, the interval(s) on which the function is increasing, decreasing, and/or constant, and its domain and range given the graph of the function? (2.3)

Ex9) Given the graph of \( f \) to the right:

a. Find the domain and the range of \( f \).
   Use interval notation.

b. Estimate all intercepts. (Write as ordered pairs)

c. Find the intervals over which \( f \) is increasing, decreasing, or constant.
   (Use interval notation for these intervals.)

d. Find the local minima and local maxima of \( f \).

Ex10) Given the graph of \( f \) to the right:

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   Use interval notation.

b. Estimate all intercepts. (Write as ordered pairs)

c. Find the intervals over which \( f \) is increasing, decreasing, or constant.
   (Use interval notation for these intervals.)

d. Find the local maxima and minima of \( f \).
Ex11) Use your calculator to graph \( f(x) = -0.4x^3 + 0.6x^2 + 3x - 2 \) over the interval \((-4, 5)\) and approximate any local maxima and local minima. Determine the intervals where the function is increasing and decreasing. Round answers to the nearest hundredth.

**H. Can you find the average rate of change of a function \( f(x) \) from \( x = a \) to \( x = b \) and interpret its meaning graphically? (2.3)**

Ex12) a. Find the average rate of change of \( f(x) = x^2 - x + 2 \) from -2 to 1.
   b. Find the equation of the secant line containing \((-2, f(-2))\) and \((1, f(1))\).
   c. Using your calculator, draw the graph of \( f(x) = x^2 - x + 2 \) and the secant line obtained in part (b) on the same screen.

**I. Can you find the difference quotient of a function? (2.1 and 2.3)**

Ex13) Find the difference quotient of \( f \), that is, find \( \frac{f(x+h)-f(x)}{h}, \ h \neq 0 \), for the following functions.

a. \( f(x) = 3x - 2 \)   
   b. \( f(x) = 2x^2 - 3x - 1 \)
   c. \( f(x) = \frac{3}{x} \)
   d. The difference quotient of \( f \) represents the slope of the ________ line containing the points \((__, ____)\) and \((__, ____\)) on the graph of the function \( y = f(x) \).

**J. Can you evaluate and graph piecewise-defined functions by hand along with finding their domains and ranges? (2.4)**

Ex14) Let \( f(x) = \begin{cases} 
-x^2 + 2 & \text{if } x < -1 \\
-2x + 1 & \text{if } x \geq -1 
\end{cases} \)

   a. Find the domain. Use interval notation.
   b. Locate any intercepts. Write them as ordered pairs.
   c. Hand sketch this function.
   d. Based on the graph, find the range. Use interval notation.
   e. Is \( f \) continuous on its domain? Explain.
Ex15) Let \( f(x) = \begin{cases} 
2x - 1 & \text{if } -3 \leq x < 1 \\
-1 & \text{if } x = 1 \\
1 + \sqrt{x} & \text{if } x > 1 
\end{cases} \)

a. Find the domain. Use interval notation.
b. Locate any intercepts. Write them as ordered pairs.
c. Hand sketch this function.
d. Based on the graph, find the range. Use interval notation.
e. Is \( f \) continuous on its domain? Explain.

K. Can you set up functions to solve maximization or minimization problems dealing with distances, areas, volumes of various geometric figures as well as real-world business situations. Can you determine where the maximum and minimum occur for these word problems using a graphing utility? (2.4 and 2.6)

Ex16) In May of 2008, a gas company had the given rate schedule for natural gas usage in single-family residences.

- Monthly service charge: $10.30
- Per therm service charge:
  - 1st 30 therms: $0.54421 per therm
  - Over 30 therms: $0.61274 per therm
- Gas charge: $0.7844 per therm.

To calculate the gas bill for a single-family residence, the company adds the service charge, the therm charge, and the gas charge.

a. Find the charge for using 30 therms in a month.
b. Find the charge for using 300 therms in a month.
c. Construct a function that relates the monthly charge \( C \) for \( x \) therms of gas.

Ex17) Let \( P = (x, y) \) be a point on the graph of \( y = x^2 - 10 \).

a. Express the distance \( D \) from \( P \) to the point \( (0, -2) \) as a function of \( x \).
b. What is \( D \) if \( x = 1 \)? What is \( D \) if \( x = -1 \)?
c. Use your graphing calculator to graph \( D \) and estimate the \( x \)-value(s) for which \( D \) is smallest. Round to the nearest hundredth.
Ex18) An island is 7 miles from the nearest point P on a straight shoreline. A town is 16 miles down the shore from P.

a. If a person can row at an average speed of 2 miles per hour and the same person can walk 3 miles per hour, express the time T that it takes to go from the island to town as a function of the distance x from P to where the person lands the boat. (Make sure that you first provide a labeled sketch for this problem.)

b. Find the domain of T. Use interval notation.

c. Use this function to determine how long it will take to travel from the island to town if the person lands the boat 4 miles from P. Round to the nearest hour.

d. Use your graphing calculator to graph T over its domain and estimate the x-value for which T is smallest. Round to the nearest tenth.

Ex19) Two cars are approaching an intersection. One is 2 miles south of the intersection and is moving at a constant speed of 30 miles per hour. At the same time, the other car is 4 miles east of the intersection and moving at a constant speed of 20 miles per hour.

a. Express the distance D between the cars as a function of the time t. (Make sure that you first provide a labeled sketch for this problem.)

b. Use your graphing calculator to graph D over its domain and estimate the t-value for which D is smallest. Round to the nearest thousandth.

Ex20) A rectangle is inscribed in a semicircle of radius 3. See the figure. Let P = (x, y) be the point in Quadrant I that is a vertex of the rectangle and is on the circle.

a. Express the area A of the rectangle as a function of x.

b. Find the domain of A. Use interval notation.

c. Use your graphing calculator to graph A over its domain and estimate the x-value for which A is largest. Round to the nearest hundredth.
Ex21) An open box with a square base is to be made from a piece of cardboard 42 inches on a side by cutting out a square from each corner and turning up the sides.

a. Express the volume $V$ of the box as a function of the length $x$ of the side of the square cut out from each corner. 
(Make sure that you first provide a labeled sketch for this problem.)

b. Find the domain of $V$. Use interval notation.

c. Use your graphing calculator to graph $V$ over its domain and estimate the $x$-value for which $V$ is largest. Round to the nearest whole number.

Ex22) An open box with a square base is required to have a volume of 10 cubic feet.

a. Express the amount $A$ of material used to make such a box as a function of the length $x$ of a side of the square base. 
(Make sure that you first provide a labeled sketch for this problem.)

b. Find the domain of $A$. Use interval notation.

c. How much material is required for a base 3 feet by 3 feet.

d. Use your graphing calculator to graph $A$ over its domain and estimate the $x$-value for which $A$ is smallest. Round to the nearest tenth.
Answers:

Ex1)
  a. Not a function as the x-value of 4 is assigned two different y-values of 2 and 5 rather than one y-value.
  b. Yes, it is a function as each distinct x-value is assigned one and only one y-value.
     Domain = \{1, 2, 3\}  Range = \{3, 5\}

Ex2)
  a. Yes, it is a function as each distinct x-value is assigned one and only one y-value.
     Domain = \((-\infty, \infty)\)
  b. No, it is not a function as \(y = \pm \sqrt{1-x}\) and thus each distinct x-value, where \(x < 1\), is assigned two y-values, not one y-value.

Ex3)
  a. \(f(-2) = \frac{3}{2}\).
  b. \(f(-x) = \frac{1-x^2}{x-4}\)
  c. \(f(x-2) = \frac{x^2-4x+3}{x+2}\)
  d. \(f(x+h) = \frac{x^2+2hx+h^2-1}{x+h+4}\)

Ex4)
  a. \(D = [4, \infty)\)
  b. \(D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)\)
  c. \(D = (4, \infty)\)
  d. \(D = (-\infty, \infty)\)

Ex5)
  a. \((f + g)(x) = x^2 + x - 12\)  Domain = \((-\infty, \infty)\)
  b. \((f - g)(x) = -x^2 + x + 6\)  Domain = \((-\infty, \infty)\)
  c. \((fg)(x) = x^3 - 3x^2 - 9x + 27\)  Domain = \((-\infty, \infty)\)
  d. \(\left(\frac{f}{g}\right)(x) = \frac{1}{x+3}\)  Domain = \((-\infty, -3) \cup (-3, 3) \cup (3, \infty)\)

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Ex6)

a. \((f + g)(x) = \frac{11x + 2}{2x - 1}\)  
   Domain = \((-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)\)

b. \((f - g)(x) = \frac{-x - 2}{2x - 1}\)  
   Domain = \((-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)\)

c. \((fg)(x) = \frac{30x^2 + 10x}{4x^2 - 4x + 1}\)  
   Domain = \((-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)\)

d. \(\left(\frac{f}{g}\right)(x) = \frac{5x}{6x + 2}\)  
   Domain = \((-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \frac{1}{2}) \cup (\frac{1}{2}, \infty)\)

Ex7)

a. \(f\) is even as its graph has y-axis symmetry.

b. \(f\) is odd as its graph has origin symmetry.

Ex8)

a. \(f\) is even as \(f(-x) = f(x)\) as \(f(-x) = 3x^4 - 5x^2\)

b. \(f\) is neither even nor odd as \(f(-x) \neq f(x)\) and \(f(-x) \neq -f(x)\) as \(f(-x) = -5x^3 + x + 2\)

c. \(f\) is odd as \(f(-x) = -f(x)\) as \(f(-x) = -1 \left(\frac{x^2 - 3}{x}\right) = -f(x)\)

Ex9)

a. Domain = \([-3, 3]\)  Range = \([-2, 2]\)

b. x-intercepts \(\approx (-0.6, 0)\) and \(\approx (0.6, 0)\)  y-intercept \(\approx (0, 2)\)

c. \(f\) is increasing on \((-2, 0) \cup (2, 3)\)
   
   \(f\) is decreasing on \((-3, -2) \cup (0, 2)\)
   
   \(f\) is never constant.

d. The local maximum value is \(f(0) = 2\).

   The local minimum value is \(f(x) = -2\) which occurs when \(x = -2\) or \(x = 2\).

Ex10)

a. Domain = \([-4, \infty]\)  Range = \((-\infty, 2]\)

b. x-intercepts \(\approx (0, 0)\) and \(\approx (3, 0)\)  y-intercept \(\approx (0, 0)\)

c. \(f\) is increasing on \((-2, 2)\)  \(f\) is decreasing on \((2, \infty)\)  \(f\) is constant on \((-4, -2)\)

d. The local maximum value is \(f(2) = 2\).

   There is no local minimum value.

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Ex11)  
The local maximum is $f(2.16) = 3.25$  
The local minimum is $f(-1.16) = -4.05$  
f is increasing on $(-1.16, 2.16)$  
f is decreasing on $(-4, -1.16) \cup (2.16, 5)$  

Ex12)  
a. The average rate of change is $-2$.  
b. The equation of the secant line is $y = 2x + 4$  

c.  

Ex13)  
a. $f(x + h) - f(x) \over h = 3$  
b. $f(x + h) - f(x) \over h = 4x + 2h - 3$  
c. $f(x + h) - f(x) \over h = -3 \over x(x + h)$  
d. $f(x + h) - f(x) \over h$ represents the slope of a secant line containing the points $(x, f(x))$ and $(x + h, f(x + h))$ on the graph of $y = f(x)$.

Ex14)  
a. Domain $= (-\infty, \infty)$  
b. x-intercepts $= (-\sqrt{2}, 0)$ and $\left(\frac{1}{2}, 0\right)$ y-intercept $= (0, 1)$  
c.  
d. Range $= (-\infty, 3]$  
e. $f$ is not continuous on $(-\infty, \infty)$ as there is a break in the function at $x = -1$.
Ex15)
a. Domain = \([-3, \infty)\)
b. \(x\)-intercepts = \(\left(\frac{1}{2}, 0\right)\) y-intercept = \((0, -1)\)
d. Range = \([-7, 1) \cup (2, \infty)\)
e. \(f\) is not continuous on \([-3, \infty)\) as there is a gap in its graph at \(x = 1\).

Ex16)
a. \(C(30) \approx 50.16\) rounded to the nearest cent.
b. \(C(300) \approx 427.39\) rounded to the nearest cent.
c. \(C(x) = \begin{cases} 10.30 + 1.32861x & \text{for } 0 \leq x \leq 30 \\ 8.24 + 1.39714x & \text{for } x > 30 \end{cases}\)
   (Note: I rounded 8.2441 to 8.24 in the above piece-wise equation.)

Ex17)
a. \(D(x) = \sqrt{x^4 - 15x^2 + 64}\) (I simplified \(\sqrt{x^2 + \left(x^2 - 10 + 2\right)^2}\))
b. \(D(1) \approx 7.07\) and \(D(-1) \approx 7.07\)
c. \(x \approx \pm 2.74\)
   Note: \(D(x)\) is an even function so it has two local minima.

Ex18)
a. \(T(x) = \frac{\sqrt{x^2 + 49} + 16 - x}{2} \div \frac{3}{2}\)
b. Domain = \([0, 16]\)
c. If you land your boat 4 miles from P, it will take you about 8 hours.
d. \(x \approx 6.3\) miles

Ex19)
a. \(D(x) = \sqrt{1300t^2 - 280t + 20}\) (I simplified \(\sqrt{(30t - 2)^2 + (20t - 4)^2}\))
b. The distance between the two cars is minimized at about \(t \approx 0.108\) hours.

Ex20)
a. \(A(x) = 2x\sqrt{9 - x^2}\)
b. Domain = \((0, 3)\)
c. The area is largest when \(x \approx 2.12\) inches

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Ex21)  
  a. \( V(x) = x(42 - 2x)^2 \)  
  b. Domain = \((0, 21)\)  
  c. The side of the square is about 7 inches.

Ex22)  
  a. \( A(x) = x^2 + \frac{40}{x} \)  
  b. Domain = \((0, \infty)\)  
  c. The surface area is approximately 22.3 square feet when the base is 3 feet by 3 feet.  
  d. \( x \approx 2.7 \) feet.