In order to prepare for a test including Sections 9.1 and 9.4, you need to understand and be able to work problems involving the following topics:

I. Polar Coordinates

A. Can you plot a point given its polar coordinates?

Ex1) Plot each point given in polar coordinates.

a. \((3, 180^\circ)\)  
b. \((-5, \frac{5\pi}{3})\)  
c. \((3, \frac{3\pi}{4})\)

B. Can you convert rectangular coordinates to polar coordinates?

Ex2) Polar coordinates for three points are given. Find the rectangular coordinates for each point.

a. \((3, 180^\circ)\)  
b. \((-5, \frac{5\pi}{3})\)  
c. \((3, \frac{3\pi}{4})\)
C. Can you convert polar coordinates to rectangular coordinates?

Ex3) The rectangular coordinates of three points are given. Find polar coordinates for each point.

a. \((0, -4)\)  
b. \((2, -2)\)  
c. \((-5\sqrt{3}, -5)\)

D. Can you transform equations from polar to rectangular form?

Ex4) The letters \(r\) and \(\theta\) represent polar coordinates. Write each equation using rectangular coordinates \((x, y)\)

a. \(r = 3\sin \theta\)  
b. \(r^2 = 4\cos \theta\)  
c. \(r = 2 + \cos \theta\)
II. Vectors

A. Do you know what scalars, vectors and unit vectors are?

Ex5) Fill in the blanks.

A vector is a quantity that has __________ and __________.

When dealing with vectors we refer to real numbers as __________.

A vector \( \mathbf{u} \) for which \( \| \mathbf{u} \| = 1 \) is called a __________ ________.

Ex6) For parts a and b below the vector \( \mathbf{v} \) has initial point \( P \) and terminal point \( Q \). Write \( \mathbf{v} \) in the form \( a \mathbf{i} + b \mathbf{j} \).

a. \( P = (3, 5) \) and \( Q = (5, 11) \)

b. \( P = (-2, 5) \) and \( Q = (-6, 8) \)

Ex7) For parts a and b below find \( \| \mathbf{v} \| \).

a. \( \mathbf{v} = -5 \mathbf{i} + 12 \mathbf{j} \)

b. \( \mathbf{v} = 2 \mathbf{i} + 2 \mathbf{j} \)

Ex8) For parts a and b below find the unit vector that has the same direction as \( \mathbf{v} \).

a. \( \mathbf{v} = -6 \mathbf{i} \)

b. \( \mathbf{v} = 8 \mathbf{i} - 15 \mathbf{j} \)
B. Can you add and subtract vectors geometrically?

Ex9) \( u = 3j \) and \( v = 2i - 2j \)

a. Graph \( u \).

b. Graph \( v \).

c. Graph \( u - v \).

d. Graph \( 2u + 3v \).

C. Can you add and subtract vectors algebraically?

Ex10) \( v = 4i - 7j \) and \( w = -2i + 6j \)

a. Find \( 3v - 5w \)

b. Find \( \|v - w\| \)

D. Can you solve applications involving vectors?

Ex11) A worker pulls a trolley with a force of 150 pounds. Then handle of the trolley makes an angle of 30° with the ground. Express the force vector in terms of \( i \) and \( j \).

Ex12) To remove a stubborn stump a homeowner and his neighbor both attach ropes from their trucks to the stump. The trucks will pull at a 45° angle from one another. The first truck pulls with 6000 pounds of force and the second truck with 5000 pounds of force. What is the resultant force on the stump?
Answers:

Ex1)

a.  

b.  

c.  

Ex2)

a.  \((-3, 0)\) 

b.  \(\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)\) 

c.  \(\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)\) 

Ex3) The answers below use \(r > 0\) and \(0 \leq \theta < 2\pi\). Other answers are possible.

a.  \(\left(4, \frac{3\pi}{2}\right)\) 

b.  \(\left(2\sqrt{2}, \frac{7\pi}{4}\right)\) 

c.  \(\left(10, \frac{7\pi}{6}\right)\) 

Ex4)

a.  \(x^2 + y^2 = 3y\)

Hint: Multiply both sides by \(r\) to make \(r \sin \theta\) on the right hand side.

b.  \((x^2 + y^2)^3 = 16x^2\)

Hint: Multiply both sides by \(r\) to make \(r \cos \theta\) on the right hand side. You may wish to square both sides of the equation when \(r\) is raised to an odd power.

c.  \((x^2 + y^2 - x)^2 = 4(x^2 + y^2)\)

Hint: Multiply both sides by \(r\) to make \(r \cos \theta\) on the right hand side. Isolate \(2r\). You may wish to square both sides of the equation when \(r\) is raised to an odd power.
Ex5) magnitude, direction, scalars, unit vector

Ex6) a. \( \mathbf{v} = 2\mathbf{i} + 6\mathbf{j} \)  b. \( \mathbf{v} = -4\mathbf{i} + 3\mathbf{j} \)

Ex7) a. \( \|\mathbf{v}\| = 13 \)  b. \( \|\mathbf{v}\| = 2\sqrt{2} \)

Ex8) a. \(-\mathbf{i}\)  b. \( \frac{8}{17}\mathbf{i} - \frac{15}{17}\mathbf{j} \)

Ex9) 

Ex10) a. \( 22\mathbf{i} - 51\mathbf{j} \)  b. \( \sqrt{205} \)

Ex11) \( 75\sqrt{3}\mathbf{i} + 75\mathbf{j} \)

Ex12) The trucks pull on the stump with a force of approximately 10,170 pounds.