Math 175: Chapter 6 Review: Trigonometric Functions

In order to prepare for a test on Chapter 6, you need to understand and be able to work problems involving the following topics.

A. Can you sketch an angle in standard position given its radian measure?
   1. Draw an angle in standard position whose measure is:
      a. \( \frac{3\pi}{4} \)  b. \( \frac{11\pi}{6} \)  c. \( -\frac{3\pi}{2} \)  d. \( \frac{10\pi}{3} \)

B. Can you find the measure of a coterminal angle for an angle whose measure is given in degrees or radians?
   2. Find a positive angle less than 360° or \( 2\pi \) that is coterminal with the given angle:
      a. 1095°  b. \( \frac{14\pi}{3} \)  c. \( -\frac{3\pi}{4} \)

C. Can you convert from degrees to radians and from radians to degrees?
   3. Convert each degree measure to an exact radian measure.
      a. -210°  b. 15°
   4. Convert each radian measure to an exact degree measure.
      a. \( \frac{4\pi}{3} \) radians  b. \( \frac{\pi}{5} \) radians
   5. Convert -51° to radian measure, rounded to two decimal places.
   6. Convert 5 radians to degree measure, rounded to two decimal places.

D. Do you know the formulas for arc length of a circle, area of a sector of a circle, linear speed and angular speed and how to use them in application problems?
   7. A neighborhood carnival has a Ferris wheel whose radius is 30 feet. You measure the time it takes for one revolution to be 70 seconds. What is the linear speed (in feet per second) of this Ferris wheel? What is the angular speed in radians per second?
   8. An engineer is asked to design a water sprinkler that will cover a field of 100 square yards that is in the shape of a sector of a circle of diameter 30 yards. Through what angle should the sprinkler rotate? Give your answer in degrees rounded to the nearest tenth of a degree.
   9. Charleston, West Virginia is due north of Jacksonville, Florida. Find the distance between Charleston (38° 21’ north latitude) and Jacksonville (30°20’ north latitude). Assume that the radius of Earth is 3960 miles. Round to the nearest mile.

E. Can you state the unit circle definition of the six trigonometric functions of any real number \( t \).
F. Can you find the exact values of \( \sin t, \cos t, \tan t, \sec t, \csc t \) and \( \cot t \) for \( t = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi \), and integer multiples of these numbers using the unit circle?

10. Using the unit circle, find the exact value of each of the following functions.
   a. \( \sin \frac{5\pi}{3} \)
   b. \( \cos \left( -\frac{5\pi}{6} \right) \)
   c. \( \tan \frac{\pi}{2} \)
   d. \( \cos 3\pi \)
   e. \( \tan \frac{5\pi}{4} \)
   f. \( 2 \sec \frac{11\pi}{3} + \tan \frac{\pi}{3} \)
   g. \( \sin \frac{11\pi}{2} \)
   h. \( \cot 7\pi \)
   i. \( \csc \left( -\frac{7\pi}{6} \right) \)

G. Can you use a calculator to find the approximate value of the trigonometric functions?

11. Use a calculator to find the approximate value of each expression rounded to two decimal places.
   a. \( \sin \frac{7\pi}{12} \)
   b. \( \tan 53^\circ \)
   c. \( \cot \frac{\pi}{8} \)
   d. \( \sec 75^\circ \)

H. Can you use a circle of radius \( r \) to evaluate the six trigonometric functions?

12. Find the exact values of each of the six trigonometric functions of an angle \( \theta \) if \((-3, 2)\) is a point on its terminal side?

I. Can you state the domain, range and period of the six trigonometric functions?

J. Can you use the Periodic Properties to find exact values of the trigonometric functions?

13. Find the exact value of:
   a. \( \cos \frac{17\pi}{4} \)
   b. \( \tan \frac{11\pi}{4} \)

K. Can you determine the signs of the trigonometric functions in a given quadrant?

14. Name the quadrant in which the angle \( \theta \) lies.
   a. \( \sin \theta < 0, \cos \theta > 0 \)
   b. \( \sec \theta < 0, \tan \theta > 0 \)

L. Can you use the Reciprocal Identities, Quotient Identities, Even-Odd Properties and the Pythagorean Identities to simplify trigonometric expressions or find exact values of the six trigonometric functions?

15. Find the exact value of each of the four remaining trigonometric function if given:
   \( \sin \theta = \frac{4}{5}, \cos \theta = -\frac{3}{5} \)
16. Find the exact value of each expression. Do not use a calculator.

a. \( \cos(-270^\circ) \)  
b. \( \cos\left(\frac{-3\pi}{4}\right) \)  
c. \( \csc\left(\frac{-\pi}{3}\right) \)  
d. \( \cot 20^\circ - \frac{\cos 20^\circ}{\sin 20^\circ} \)  
e. \( \sec\left(\frac{-\pi}{18}\right) \cdot \cos \frac{37\pi}{18} \)  
f. \( \frac{\sin 60^\circ}{\cos(-420^\circ)} - \tan(-60^\circ) \)

17. If \( \cos \theta = 0.2 \), find the value of \( \cos \theta + \cos(\theta + 2\pi) + \cos(\theta + 4\pi) \)

M. Can you find the exact value of the remaining five trigonometric functions given one of the functions and the quadrant of the angle?

18. Find the exact value of each of the remaining five trigonometric functions of \( \theta \) if given:

a. \( \cos \theta = \frac{\sqrt{3}}{3} \) and \( \frac{3\pi}{2} < \theta < 2\pi \).  
b. \( \cot \theta = \frac{5}{4}, \sec \theta < 0 \)

N. Can you graph one cycle of \( y = \sin x, y = \cos x, y = \tan x, y = \sec x, y = \csc x \) and \( y = \cot x \) quickly by hand, using key points, without a table of values?

O. Can you graph transformations for the six trigonometric functions by hand and state their periods, amplitudes and phase shifts if applicable?

19. Find amplitude, period and phase shift. Then graph two cycles of each function by hand. Use the graph to find the domain and range of each function.

a. \( y = -2\sin(3x - \pi) \)

b. \( y = \frac{1}{2} \cos\left(\frac{1}{2}x + \frac{\pi}{4}\right) \)

c. \( y = \sin\left(\pi x - \frac{\pi}{2}\right) + 1 \)

20. Graph two cycles of the function \( y = \tan\left(\frac{1}{2}x\right) - 1 \) by hand. Use the graph to determine the domain and range of the function.

P. Can you write the equation of a sine function given amplitude, period and phase shift?

21. Write the equation of a sine function that has the given characteristics.

Amplitude: 3;  Period: \( \frac{\pi}{2} \);  Phase shift: 2
Q. Can you find the equation for a sinusoidal graph?

22. Find an equation for each graph.

a. 

![Sinusoidal Graph](image)

b. 

![Sinusoidal Graph](image)

R. Can you build sinusoidal models from data?

23. The following data represent the average monthly temperatures for Washington, D.C.

<table>
<thead>
<tr>
<th>Month, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp. °F</td>
<td>34.6</td>
<td>37.5</td>
<td>47.2</td>
<td>56.5</td>
<td>66.4</td>
<td>75.6</td>
<td>80.0</td>
<td>78.5</td>
<td>71.3</td>
<td>59.7</td>
<td>49.8</td>
<td>39.4</td>
</tr>
</tbody>
</table>

a. Draw a scatter gram of the data for one period.
b. Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.
c. Draw the sinusoidal function found in part (b) on the scatter diagram.
d. Use a graphing utility to find the sinusoidal function of best fit.
e. Graph the sinusoidal function of best fit on a scatter diagram of the data.
Answers:

1. 
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 
   g. 
   h. 

2. 
   a. 15° 
   b. \( \frac{2\pi}{3} \) 
   c. \( \frac{5\pi}{6} \) 
   d. \( -\frac{7\pi}{6} \) 
   e. \( \frac{\pi}{12} \) 

4. 
   a. 240° 
   b. 36° 

5. 
   a. \(-0.89\) radians 
   b. 286.48° 

7. linear speed = \( \frac{6\pi}{7} \) feet per second, angular speed = \( \frac{\pi}{35} \) radians per second 

8. 50.9° 

9. 554 miles 

10. 
    a. \( -\frac{\sqrt{3}}{2} \) 
    b. \( -\frac{\sqrt{3}}{2} \) 
    c. undefined 
    d. -1 
    e. 1 
    f. \( 4 + \sqrt{3} \) 
    g. -1 
    h. undefined 
    i. 2 

11. 
    a. 0.97 
    b. 1.33 
    c. 2.41 
    d. 3.86 

12. \( \sin \theta = \frac{2\sqrt{13}}{13}, \cos \theta = -\frac{3\sqrt{13}}{13}, \tan \theta = -\frac{2}{3}, \sec \theta = -\frac{\sqrt{13}}{3}, \csc \theta = \frac{\sqrt{13}}{2}, \cot \theta = \frac{3}{2} \) 

13. 
    a. \( \frac{\sqrt{2}}{2} \) 
    b. -1 

14. a. Quadrant IV 
    b. Quadrant III 

15. \( \csc \theta = \frac{5}{4}, \sec \theta = -\frac{5}{3}, \tan \theta = -\frac{4}{3}, \cot \theta = -\frac{3}{4} \) 

16. 
    a. 0 
    b. \( -\frac{\sqrt{2}}{2} \) 
    c. \( -\frac{2\sqrt{3}}{3} \) 
    d. 0 
    e. 1 
    f. 2\sqrt{3} 

17. 0.6 

18. 
    a. \( \sin \theta = -\frac{\sqrt{6}}{3}, \sec \theta = \sqrt{3}, \csc \theta = -\frac{\sqrt{6}}{3}, \tan \theta = -\frac{\sqrt{2}}{2}, \cot \theta = -\frac{\sqrt{2}}{2} \) 
    b. \( \tan \theta = \frac{4}{5}, \sec \theta = -\frac{\sqrt{41}}{5}, \cos \theta = -\frac{5\sqrt{41}}{41}, \sin \theta = -\frac{4\sqrt{41}}{41}, \csc \theta = -\frac{\sqrt{41}}{4} \) 

19. a. Domain: \(( -\infty, \infty )\) 
   
   Range: \([-2, 2]\) 
   
   Amplitude = 2 
   
   Period = \( \frac{2\pi}{3} \) 
   
   Phase Shift: \( \frac{\pi}{3} \)
b. Domain: \((-\infty, \infty)\)

Range: \([-\frac{1}{2}, \frac{1}{2}]\)

Amplitude = \(\frac{1}{2}\)

Period = \(4\pi\)

Phase Shift: \(-\frac{\pi}{2}\)

\[
\begin{align*}
\text{Domain: } & (-\infty, \infty) \\
\text{Range: } & \left[-\frac{1}{2}, \frac{1}{2}\right] \\
\text{Amplitude: } & \frac{1}{2} \\
\text{Period: } & 4\pi \\
\text{Phase Shift: } & -\frac{\pi}{2}
\end{align*}
\]

\[
\begin{align*}
\text{Domain: } & (-\infty, \infty) \\
\text{Range: } & [0, 2] \\
\text{Amplitude: } & 1 \\
\text{Period: } & 2 \\
\text{Phase Shift: } & \frac{1}{2}
\end{align*}
\]

20. Domain: \(\{x | x \neq k\pi, k \text{ is an odd integer}\}\)

Range: \((-\infty, \infty)\)
21. \( y = \pm 3 \sin(4x - 8) \)

22. a. \( y = 4 \sin \left( \frac{1}{4} x \right) \)  
    b. \( y = -\frac{5}{2} \cos(\pi x) \)

23. a. 

\[
\begin{align*}
\text{b.} \quad & y = 22.7 \sin \left( \frac{\pi}{6} x - \frac{2\pi}{3} \right) + 57.3
\end{align*}
\]

\[\begin{align*}
\text{c.}
\end{align*}\]

\[\begin{align*}
\text{d.} \quad & y = 22.61279198 \sin(0.5031679077x - 2.038371236) + 57.16859907
\end{align*}\]

\[\begin{align*}
\text{e.}
\end{align*}\]