CONSECUTIVE INTEGER & NUMBER PROBLEMS

Before you begin working on this introduction to Number Problems, it may be helpful to review the handout on translating sentences into algebraic forms.

The following is a table of some commonly used key words or phrases and their corresponding math symbol(s):

<table>
<thead>
<tr>
<th>Key Words</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>is / is equal to / is the same as / gives / amounts to / results in</td>
<td>=</td>
</tr>
<tr>
<td>add / added to / sum / the sum of / plus / increased by / more than / combined with</td>
<td>+</td>
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<tr>
<td>subtract / minus / difference / the difference between / subtract from / decreased by / less than / reduced by</td>
<td>-</td>
</tr>
<tr>
<td>multiply / product / times / multiplied by</td>
<td>×</td>
</tr>
<tr>
<td>quotient / divided by / per / over / ratio</td>
<td>÷</td>
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To solve word problems, follow this outline:

1. Read the problem carefully. You may have to read it over several times to understand what is given and what you are asked to do.
2. Identify the unknown number or numbers. If two or more numbers are to be found, represent one of the number using a suitable letter, such as x or n, and then represent the other number or numbers in terms of x. For example, if a second number were twice as large as the first, the numbers can be represented by x and 2x respectively.
3. Write an equation involving the unknown(s).
4. Solve the equation of the unknown(s)
5. Check your answer(s) in the equation.
6. State your answer(s) using complete sentences.
The following are examples involving one unknown number:

Ex. 1: Three times a number decreased by 7 is equal to 32. Find the number.

Step 1: Read the problem to understand it completely
Step 2: Assign a variable to the unknown:
Let \( x \) be the number.

Step 3: Set up an equation
Three times a number decreased by 7 is equal to 32
\[
3x - 7 = 32
\]

Step 4: Solve the equation
\[
\begin{align*}
3x - 7 &= 32 \\
3x - 7 + 7 &= 32 + 7 \\ 
3x &= 39 \\
\frac{3x}{3} &= \frac{39}{3} \\
x &= 13
\end{align*}
\]

Step 5: Check the solution in the equation
\(3 \cdot 13 - 7 = 32\) (True statement)

Step 6: State the answer using words.
The number is 13.

Ex. 2: One-third of a number increased by twice the same number is 35. Find the number.

Step 1: Read the problem to understand it completely
Step 2: Assign a variable to the unknown
Let \( x \) be the number.

Step 3: Set up an equation
One-third of a number increased by twice the same number is 35
\[
\frac{1}{3} x + 2x = 35
\]

Step 4: Solve the equation
\[ \frac{1}{3} x + 2x = 35 \]
\[ \frac{1}{3} \left( \frac{1}{3} x + 2x \right) = 3 \cdot 35 \] Multiplt both sides by 3
\[ x + 6x = 105 \] Use distributive property

\[ 7x = 105 \] Combine like terms
\[ \frac{7x}{7} = \frac{105}{7} \] Divide by 7
\[ x = 15 \]

**Step 5: Check the solution in the equation**
\[ \frac{15}{3} + 2 \cdot 15 = 35 \] (True statement)

**Step 6: State the answer using words**
The number is 15.

Ex. 3: Four times a number minus 9 equals five times the sum of the number and three. Find the number.

**Step 1: Read the problem to understand it completely**

**Step 2: Assign a variable to the unknown**
Let \( x \) be the number.

**Step 3: Set up an equation**
4 times a number minus 9 equals 5 times the sum of the number and 3.
\[ 4 \cdot x - 9 = 5 \cdot (x + 3) \]

**Step 4: Solve the equation**
\[ 4x - 9 = 5(x + 3) \]
\[ 4x - 9 = 5x + 15 \] Use distributive property
\[ 4x - 5x - 9 = 5x - 5x + 15 \] Subtract 5x from both sides
\[ -x - 9 = 15 \]
\[ -x - 9 + 9 = 15 + 9 \] Add 9 to each side
\[ -x = 24 \]
\[ x = -24 \] Divide both sides by -1
7x = 105
\[ \frac{7x}{7} = \frac{105}{7} \]
\[ x = 15 \]

**Step 5: Check the solution in the equation**

4 \cdot (-24) - 9 = 5 \cdot (-24 + 3) \text{ (True statement)}

**Step 6: State the answer using words.**
The number is -24.

The following examples illustrate problems involving more than one unknown.

Ex. 4: The sum of three numbers is 59. The second number is twice the first and the third is 4 more than the second. Find the numbers.

**Step 1: Read the problem to understand it completely**

**Step 2: Assign variable names to the unknowns**

Let x be the first number, 2x be the second number, and 2x + 4 be the third number.

**Step 3: Set up an equation**

The sum of the three numbers is 59.

1st number + 2nd number + 3rd number = 59

x + (2x) + (2x + 4) = 59

**Step 4: Solve the equation**

x + 2x + 2x + 4 = 59

5x + 4 = 59 \text{ Combine like terms}

5x + 4 - 4 = 59 - 4 \text{ Subtract 4 from both sides}

5x = 55 \text{ Divide by 5}

x = 11

**Step 5: Check the solution in the equation**

11 + 2 \cdot 11 + (2 \cdot 11 + 4) = 59 \text{ (True statement)}

**Step 6: State the answer using words**

The first number is 11, the second number is 22, and the third number is 26.
Ex. 5: The difference between 2 numbers is 16. The larger number is 25 more than one-half of the smaller number. What are the numbers?

Step1: Read the problem to understand it completely
Step 2: Assign variable names to the unknowns
Let x be the smaller number, and \( \frac{x}{2} + 25 \) be the larger number.

Step 3: Set up an equation
The difference between the numbers is 16
larger number - smaller number is 16

\[ \frac{x}{2} + 25 - x = 59 \]

Step 4: Solve the equation
\[ \frac{x}{2} + 25 - x = 16 \]
\[ 2 \cdot \left( \frac{x}{2} + 25 - x \right) = 16 \cdot 2 \quad \text{Multiply both sides by 2} \]
\[ x + -2x + 50 = 32 \quad \text{Combine like terms} \]
\[ -x + 50 = 32 \]
\[ -x + 50 - 50 = 32 - 50 \quad \text{Subtract 50 from both sides} \]
\[ -x = -18 \]
\[ -1 \cdot -x = -1 \cdot -18 \quad \text{Multiply both sides by -1} \]
\[ x = 18 \]
\[ \frac{x}{2} + 25 = \frac{18}{2} + 25 = 9 + 25 = 35 \]

Step 5: Check the solution in the original equation
\[ \frac{18}{2} + 25 - 18 = 16 \quad \text{(True statement)} \]

Step 6: State the answer using words
The smaller number is 18 and the larger number is 34.

Ex. 6: The difference between two numbers is nine. The larger number is five less than three times the smaller number. Find the numbers.

Step1: Read the problem to understand it completely
Step 2: Assign variable names to the unknowns
Let x be the smaller number and \( 3x - 5 \) be the larger number.
Step 3: Set up an equation
The difference between the numbers is 9
larger number - smaller number is 9

\[(3x - 5) - x = 9\]

Step 4: Solve the equation

\[3x - 5 - x = 9\] Combine like terms
\[2x - 5 + 5 = 9 + 5\] Add 5 to both sides
\[2x = 14\] Divide by 2
\[x = 7\]

Step 5: Check the solution in the original equation
\[3\cdot 7 - 5 - 7 = 9\] (True statement)

Step 6: State the answer using words
The smaller number is 7 and the larger number is 16.

Consecutive Integers

Consecutive integers are integers that differ by one. For example, 8, 9, and 10 are consecutive integers; so are -5, -4, and -3. If x represents an integer, then x + 1 is the next consecutive integer and x + 2 is the next consecutive integer right after x + 1.

Consecutive even integers differ by 2. For example, 8, 10, and 12 are consecutive even integers. If x is an even integer, then x + 2 is the next even consecutive integer, and x + 4 is the next one right after x + 2.

Consecutive odd integers differ by 2. For example 9, 11, 13, and 15 are consecutive odd integers. If x is an odd integer, then x + 2, x + 4, and x + 6 are the next three consecutive odd integers.

Ex. 7: The sum of three consecutive integers is 102. What are the integers?
Step 1: Read the problem to understand it completely
Step 2: Assign variable names to the unknowns
Let x be the first integer, x + 1 be the second integer, and x + 2 be the third integer.
Step 3: Setup an equation
1st integer + 2nd integer + 3rd integer = 102
\[x + (x+1) + (x + 2) = 102\]
Step 4: Solve the equation
\[ x + (x + 1) + (x + 2) = 102 \]
\[ 3x + 3 = 102 \quad \text{Combine like terms} \]
\[ 3x + 3 - 3 = 102 - 3 = 99 \quad \text{Subtract 3 from both sides} \]
\[ \frac{3x}{3} = \frac{99}{3} \quad \text{Divide each side by 3 to solve for } x \]
\[ x = 33 \]
\[ x + 1 = 34 \]
\[ x + 2 = 35 \]

Step 5: Check your answer
\[ 33 + (33 + 1) + (33 + 2) = 102 \text{ (True statement)} \]

Step 6: Write your answers in words
The first integer is 33, the second integer is 34, and the third integer is 35.

Ex. 8: The sum of three consecutive odd integers is – 69. What are the integers?

Step 1: Read the problem to understand it completely
Step 2: Assign variable names to the unknowns
Let \( x \) be the first integer, \( x + 2 \) be the second consecutive odd integer, and \( x + 4 \) be the third consecutive odd integer.

Step 3: Setup an equation
\[ 1^{\text{st}} \text{ integer} + 2^{\text{nd}} \text{ integer} + 3^{\text{rd}} \text{ integer} = -69 \]
\[ x + (x + 2) + (x + 4) = -69 \]

Step 4: Solve the equation
\[ x + (x + 2) + (x + 4) = -69 \]
\[ 3x + 6 = -69 \quad \text{Combine like terms} \]
\[ 3x + 6 - 6 = -69 - 6 = -75 \quad \text{Subtract 3 from both sides} \]
\[ \frac{3x}{3} = \frac{-75}{3} \quad \text{Divide each side by 3 to solve for } x \]
\[ x = -25 \]
\[ x + 2 = -23 \]
\[ x + 4 = -21 \]

Step 5: Check your answer
\(-25 + (-25 + 2) + (-25+4) = -69 \text{ (True statement)} \)
Step 6: Write your answers in words
The first odd integer is -25, the second consecutive odd integer is -23, and the third consecutive odd integer is -21.

Ex. 9: Find three consecutive even integers such as the sum of the first two integers is 72 more than the third integer.

Step 1: Read the problem to understand it completely.
Step 2: Assign variable name to each of the unknowns.
Let $x$ be the first integer, $x + 2$ be the second consecutive even integer, and $x + 4$ be the third consecutive even integer.

Step 3: Setup an equation
$1^{st}$ integer + $2^{nd}$ integer = 72 more than $3^{rd}$ integer
$x + (x + 2) = 72 + (x + 4)$

Step 4: Solve the equation
$x + (x + 2) = 72 + (x + 4)$

$2x + 2 = 76 + x$ Combine like terms
$2x - x + 2 = 76 + x - x$ Subtract $x$ from each side
$x + 2 = 76 - 2$ Subtract 2 from each side
$x = 74$
$x + 2 = 76$
$x + 4 = 78$

Step 5: Check your answer
$74 + (74 + 2) = 72 + (74 + 4)$ (True statement)

Step 6: Write your answers in words
The first integer is 74, the second integer is 76, and the third integer is 78.

Ex. 10: A certain number is three less than the first of three consecutive integers. Three times this number is 11 more than the third integer. Find the number

Step 1: Read the problem to understand it completely
Step 2: Assign a variable name to each of the unknowns
Let $x$ be the number.
Since the first consecutive integer is 3 more than the number, then $x + 3$ is the first consecutive integer.
$x + 4$ is the second consecutive integer and the $x + 5$ is the third consecutive integer.

Step 3: Setup an equation
Three times this number is 11 more than the last consecutive integer.
$3x = 11 + (x + 5)$
Step 4: Solve the equation
\[3x = 11 + (x + 5)\]
Combine like terms
\[3x = 16 + x\]
Subtract x from each side
\[3x - x = 16 + x - x\]
Divide both sides by 2
\[2x = 16\]
\[x = 8\]

Step 5: Check your answer
\[3(8) = 11 + (8 + 5)\] (True statement)

Step 6: Write your answers in words
The number is 8.

Another way to solve this problem.
Let x be the first consecutive even integer, x + 1 is the second integer, and x + 2 is the third one. Since the unknown number is three less than the first of the three consecutive integers, the number can be represented by \(x - 3\).
Three times this number is 11 more than the last integer.
\[3(\text{x-3}) = 11 + (x + 2)\]
Combine like terms and distribute
\[3x - 9 = 13 + x\]
Subtract x from each side
\[3x - 9 - x = 13 + x - x\]
Add 9 to both sides
\[2x = 22\]
Divide by 2
\[x = 11\]
The number is \(11 - 3 = 8\).

Exercises: Set up and solve an equation for each problem. Follow the steps of problem solving.

Number Problems Involving One Unknown

1. Three less than twice a number is equal to the number. What is the number?

2. Twice a certain number, increased by five, is the same as three times the number. What is the number?

3. Five times a number is 12 more than three times the number. Find the number.

4. Twice a certain number is equal to the sum of six times the number and 28. Find the number.
5. Twice the sum of a number and six is the same as increasing the number by 28. Find the number.

6. When three times the sum of a number and four is added to seven times the number, the result is the same as 12 times the number. Find the number.

7. You may use your calculator to solve this problem after you set up the equation. When a number is decreased by 5.67, and this difference is multiplied by 11.05, the result is the same as twice the number increased by 0.607. Find the number.

8. When a number is decreased by 4.232, the result is the same as increasing twice the number by 0.815. Find the number. You may use your calculator.

Number Problems Involving More Than One Unknown

9. A second number is five larger than the first, and the sum of these numbers is 31. What are the numbers?

10. The second of two numbers is two less than three times the first. Find these numbers if their sum is 50.

11. One number is ten more than twice a second number. If their sum is four, find the number.

12. One of two numbers is 14 more than twice another number. When these numbers are added, the sum is 68. Find both numbers.

13. The sum of two numbers is 52. If 3 times the smaller number is one more than twice the larger number, find the two numbers.

14. The first of two numbers is three less than the second. When twice the first number, increased by five, is added to six times the second, the sum will equal 95. Find the numbers.

15. The sum of three numbers is 95. The second number is twice the first, and the third number is five more than the second. Find the numbers.

16. One number is twice a second number. When double the first number, increased by 15, is added to triple the second number, the sum is equal to one. Find the numbers.

Consecutive integer Problems

17. Find two consecutive integers whose sum is 53.

18. Find two consecutive even integers whose sum is 70.

19. The sum of two consecutive odd integers is 40. What are the integers?

20. Find three consecutive integers whose sum is 60.
21. The sum of three consecutive odd integers is 117. Find the integers.

22. If the sum of three consecutive even integers is increased by 30, the result is six. Find these integers.

23. If the sum of four consecutive even integers is -4, what are the four integers?

24. Find two consecutive integers such that the twice the first plus three times the second is equal to 53.

25. Can you find three consecutive odd integers such that the sum of the first and the second is 19 more than three times the third?

26. The largest of four consecutive odd integers is three times the smallest. Find the integers.

27. Find three consecutive odd integers such that twice the sum of the two smaller ones is 17 more than three times the largest one.

28. A certain number is six more than the largest of three consecutive integers. Twice this number is nine less than the sum of the three integers. Find the number.

29. When the sum of three consecutive integers is divided by 15, the quotient is six. Find the integers.

30. When the sum of three consecutive even integers is divided by 6, the quotient is 11. Find the integers.

31. In the set of integers 1, 5, 9, 13, 17, ..., there are three integers in a row whose sum is 99. Which three are they?
Solutions to Odd Numbered Exercises & Answers to Even Numbered Exercises

1. Let $x$ be the number.
   \[2x - 3 = x\]
   \[x = 3\]
   The number is 3.

2. The number is 5

3. Let $x$ be the number.
   \[5x = 3x + 12\]
   \[2x = 12\]
   \[x = 6\]
   The number is 6.

4. The number is -7.

5. Let $x$ be the number.
   \[2(x + 6) = x + 28\]
   \[2x + 12 = x + 28\]
   \[x = 16\]
   The number is 16

6. The number is 6.

7. Let $x$ be the number.
   \[11.05(x - 5.76) = 2x + 0.607\]
   \[11.05x - 63.648 = 2x + 0.607\]
   \[9.05x = 64.225\]
   \[x = 7.1\]
   The number is 7.1

8. The number is -5.047.

9. Let $x$ be the first number and $x + 5$ be the second number.
   \[x + (x + 5) = 31\]
   \[2x + 5 = 31\]
   \[2x = 26\]
   \[x = 13\]
   The numbers are 13 and 18.

10. The numbers are 13 and 37.
11. Let \( x \) be the second number and \( 2x + 10 \) be the first number.  
\[
x + (2x + 10) = 4
\]
\[
3x + 10 = 4
\]
\[
3x = -6
\]
\[
x = -2
\]
\[
2x + 10 = 2 \cdot (-2) + 10 = -4 + 10 = 6
\]
The numbers are -2 and 6.

12. The numbers are 18 and 50.

13. Let \( x \) be the larger number and \((2x + 1) / 3\) be the smaller number.  
\[
x + \frac{2x + 1}{3} = 52
\]
\[
3x + 2x + 1 = 156
\]
\[
5x + 1 = 156
\]
\[
5x = 155
\]
\[
x = 31
\]
\[
\frac{2x + 1}{3} = \frac{2 \cdot 31 + 1}{3} = \frac{63}{3} = 21
\]
The larger number is 31 and the smaller number is 21.

14. The numbers are 9 and 12.

15. Let \( x \) be the first number, \( 2x \) be the second number, and \( 5 + 2x \) be the third number.  
\[
x + (2x) + (5 + 2x) = 95
\]
\[
5x + 5 = 95
\]
\[
5x = 90
\]
\[
x = 18
\]
The numbers are 18, 36, and 41.

16. The numbers are -4 & -2.

17. Let \( x \) be the first integer and \( x + 1 \) be the second integer.  
\[
x + (x + 1) = 53
\]
\[
2x + 1 = 53
\]
\[
2x = 52
\]
\[
x = 26
\]
The integers are 26 and 27.
18. The integers are 34 & 36.

19. Let \( x \) be the first odd integer and \( x + 2 \) be the second consecutive odd integer.
   
   \[
   x + (x + 2) = 40 \\
   2x + 2 = 40 \\
   2x = 38 \\
   x = 19
   \]
   
   The integers are 19 and 21.

20. The integers are 19, 20, and 21.

21. Let \( x \) be the first consecutive odd integer, \( x + 2 \) be the second one, and \( x + 4 \) be the third integer.
   
   \[
   x + (x + 2) + (x + 4) = 117 \\
   3x + 6 = 117 \\
   3x = 111 \\
   x = 37
   \]
   
   The integers are 37, 39, and 41.

22. The integers are -10, -8, and -6.

23. Let \( x \) be the first consecutive even integer, \( x + 2 \) be the second one, \( x + 4 \) be the third, and \( x + 6 \)
    be the fourth one.
   
   \[
   x + (x + 2) + (x + 4) + (x + 6) = -4 \\
   4x + 12 = -4 \\
   4x = -16 \\
   x = -4
   \]
   
   The integers are -4, -2, 0, and 2.

24. The integers are 10 and 11.

25. Let \( x \) be the first odd integer, \( x + 2 \) be the second integer, and \( x + 4 \) be the third one.
   
   \[
   x + (x + 2) = 19 + 3(x + 4) \\
   2x + 2 = 19 + 3x + 12 \\
   2x + 2 = 31 + 3x \\
   -x = 29 \\
   x = -29
   \]
   
   The integers are -29, -27, and -25.

26. The integers are 3, 5, 7, and 9.
27. Let \( x \) be the first consecutive odd integer, \( x + 2 \) be the second integer, and \( x + 4 \) be the third one.

\[
2(x + x + 2) = 17 + 3(x + 4)
\]
\[
4x + 4 = 17 + 3x + 12
\]
\[
4x + 4 = 3x + 29
\]
\[
x = 25
\]

The integers are 25, 27, and 29.

28. The number is 30

29. Let \( x \) be the first consecutive integer, \( x + 1 \) is the second integer, and \( x + 2 \) is the third one.

\[
\frac{x + (x + 1) + (x + 2)}{15} = 6
\]
\[
3x + 3 = 90
\]
\[
3x = 87
\]
\[
x = 29
\]

The integers are 29, 30, and 31.

30. The integers are 20, 22, and 24.

31. Let \( x \) be the first integer, \( x + 4 \) be the second integer, and \( x + 8 \) be the third integer.

\[
x + (x + 4) + (x + 8) = 99
\]
\[
3x + 12 = 99
\]
\[
3x = 87
\]
\[
x = 29
\]

The integers are 29, 33, and 37.

To learn more about solving word problems involving consecutive integers, visit the website below. The website contains exercises with worked solutions and videos.

http://www.onlinemathlearning.com/consecutive-integer-problems.html

To learn more about solving number word problems, refer to the website below. The website has many exercises with worked solutions.

http://www.purplemath.com/modules/numbprob.htm